Design and optimization of planetary gears under consideration of all relevant influences

Prof. Dr.-Ing. Berthold Schlecht, TU Dresden
Dr.-Ing. Tobias Schulze, DriveConcepts GmbH Dresden

Abstract

The calculation of gears especially planetary gears can just be carried out by the consideration of influences of the whole drive train and the analysis of all relevant machine elements. In this case the gear is more than the sum of its machine elements. Relevant interactions need to be considered under real conditions. The standardized calculations are for the safe dimensioning of the machine elements with the consideration of realistic load assumptions decisive. But they need to be completed by extended analysis of load distribution, flank pressure, root stress, transmission error and contact temperature, /1,2,4/.

Gear design process

The need of light weight construction and taking resources into consideration results into gearbox designs with high load capacity and power density. At the same time we have very high expectations for reliability of the gear. Additional there is a diversity of planetary gears for different application cases. Gears with one or more stages, with one or more gearbox inputs and outputs are realized. Furthermore different kind of toothings exists: spur and helical gears and also double helical gears are possible. For mounting of shafts and gearings roller bearings and sliding bearing are used, Fig. 1.

All these condition result in special and additional design criteria with consideration of maximum of load, in addition dynamic loads under different load situations. Experiences from drive drains with stiff foundations and constant external loads are not applicable directly, because of the unique boundary conditions, dynamic excitation of the structure, changing influences by external conditions, /12/.

![A. Lange & Söhne Chronometer](image1)
![Rohloff SPEEDHUB 500/14 Internal gear hub](image2)
![Ferrari F430 Formula 1 gearbox](image3)

Fig. 1: Application of planetary gears /13/
The product design process of a gear begins classically with the load calculation, followed by gear and component layout to the point of structure analysis, Fig. 2.

Only at a test bench or in industrial use as a component in the whole drive train, can the quasi static and dynamic behavior of the gear in actual conditions be verified. This long chain in the process does not allow an efficient gear calculation, especially considering the insecurities of the load assumptions, and with that the inevitable inaccurate stress of the single machine elements and the resulting strains.

Fig. 2:  Classic product design process

In these cases the highly precise and in part standardized calculations of machine elements can only be applicable, as far as the accuracy of the load assumptions allow. Any interactions of the single elements within the stressed gear (for example the influence of axel bending on the load dispersion of the gearing) are thereby lost. Furthermore the gear must – especially by flexible fundaments or dynamic excitation – be understood as a sub-system of the drive train. Only this way can a realistic load gradient be constructed, /13/.

An evenly balanced calculation model for drive trains, which connects all concerned sub disciplines (external conditions, drive train dynamics, structure dynamics, electrical phenomena and machine regulation) in a comparative model depth, is missing, Fig. 3. Only such a balanced model that allows for all needed conditions, can deliver the realistic and reliable statements on dynamic strains needed to make the safe design of drive components possible, /15/.

Fig. 3:  Design process of a gear as a system
The resulting problems and damages can not only be explained through analysis of the single modules. In fact the essential influences of the surrounding system components must be accounted for and included in the computation. Here arises the real difficulty of finding the necessary system parameters to solve the respective question.

That is why the product development process of the future is moving more and more to system analysis, instead of the design of single machine elements. Decisive in gear development is the continuous – mostly software supported – analysis, result conditioning and data maintenance to the point of supervision of the life cycle of a gear. On the one side all calculations of the machine elements gear, axle, bearing, axle-hub connection, screw connection etc. are to be implemented following the current standards. These must be supplemented through detailed examination of load gradients, load distribution, to the point of optimization of single target parameters (mass, stiffness, ..)

**Gearbox development and calculation according standards**

Especially for design concepts of planetary and spur gearboxes the newest development of DriveConcepts GmbH - the product MDESIGN® gearbox - is established. This calculation software gives complete product information in the early phase of product life cycle (PLC). The calculation can’t replace measurements and test drives, but iteration steps can be reduced economically. The software allows an intuitive and easy handling in the design process of whole gearboxes from the dimensioning of the machine elements – shafts, bearings and toothings - according to the actual standards, /5,6,7/.

For toothing are implemented:
- DIN 3990:1987 T1-T6

Future work for toothings:
- micro pitting according to ISO/TR 15144-1
- scuffing according to ISO/TR 13989 1 & 2, AGMA 925
- gear mesh efficiency / loss factor H_V & H_VL

The shafts of the gearbox are calculated according to:
- DIN 743:2008 T1-T4 & Beiblatt 1,2

For the roller bearings are different calculations possible:
- life time L_{H10} according DIN ISO 281:2009
- modified life time according DIN ISO 281:2009 Beiblatt 1,3
- advanced modified life time according DIN ISO 281:2009 Beiblatt 1,3
- life time according ISO/TR 16281:2009

So the software allows calculating the system gearbox in one step including a complete documentation into a PDF/A document according to ISO 19005-1:2005, Fig. 4.
Gear optimization (macro geometry)

The following chapter shows the gear optimization at some case studies:

Load distribution

Next to the load distribution factor $K_H$, it is one of the important tasks of gear development to optimize the load distribution of each planet gear. This is done using a pure statistic model, which determines the load distribution factor $K_H$. The load distribution factor is defined as the ratio of the maximum tooth normal force to the median tooth normal force at the speed of zero. Dynamic factors are represented by the factor $K_v$. The median contact stiffness from the load gradient calculation is used for the analysis, as well as the wheel body stiffnesses (sun, ring gear), the bearing stiffness and the bearing clearances (sun, planet, ring gear and planet carrier). The following deviations can be accommodated, see Fig. 5.

- Single pitch deviation sun and ring gear
- Tooth width variations planet gear
- Center distance deviation and planet carrier pitch deviations
- Displacement sun, planet carrier, ring gear

Fig. 5: Computation model for load distribution $K_H$
The computation of the load distribution allows statements on suitable tolerances or tolerable location variations with exact knowledge of the real load for every single planet. These investigations allow for example single parameters to be analyzed with regard to their influence on the load bearing capacity of the gearing, see Fig. 6.

Suitable construction parameters as well as sensible tolerances for gearings and location variations can be defined. Research on load distribution ($K_\text{fl}$ and $K_\text{pl}$) present distributed gears have shown that only a simultaneous optimization of load distribution on flank ($K_\text{fl}$) and to planets ($K_\text{pl}$) results in an optimal gear, Fig. 7. An effective instrument for a balanced load distribution is the use of optimized flexible planet gear bearings. The impact is due to the targeted overlapping of bolt and bushing bending with the goal of minimizing the tilt angle of the planet, which is determined by the deformation of the bushing, /16/.

**Stiffness optimization**

The optimization of construction parameters with the goal of an optimal stiffness of all relevant gear elements is probably one of the most complex development tasks in the design.
process. Exemplary is the description of the following variation analysis on a planet mount. The goal is a design with the least possible mass while necessary stiffness requirements needed in view of the load gradient, Picture 12. Both one sided as well as two sided samples can be considered. These can be constructed with a round or optimized outline (triangular, square), Fig.8.

![Variant of planet carriers: single plate (left) - double plate (right) /3/](image)

The geometric parameters to be varied in such a study are shown in Fig.9. Through the large amount of parameters it is necessary to use software programs with integrated FE-solvers for calculating stiffness parameters. Only on this way can optimal configurations be found for the whole parameter area.

![Geometric parameters of planet carriers: single plate (left) - double plate (right)](image)

**Mass-/design space optimization**

Not only has the just introduced stiffness optimization led the engineer to a number of detail problems. The search for a mass and construction size optimized gear is a highly complex question, due to the number of overlapping influences. Fig. 10 shows the field of results of a variation study for a constant given total gear ratio and a defined load.
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Dr.-Ing. Tobias Schulze – DriveConcepts GmbH, Dresden

Fig. 10: Optimization mass and design space
The investigation can be used to improve present gear solutions, as well as for new designs. Using an existing design as an example, in the following it is shown how high the potential can be, Fig. 11.

Fig. 11: Variation study mass optimization: initial state (left) – mass optimized gear (right)

At similar dimensions for the ring gear outer diameter $d_3$ of gear stage 2, one arrives at a mass saving by adjusting the ring gear diameter for stage 1 and reducing the tooth width. MDESIGN® gearbox avoids the over dimensioning of planet gears by presetting safety factors for the gearbox machine elements. The mass of the original is at $m_{ges}=2200$ kg. All generated optimized solutions arrive at a mass reduction in comparison to the real life gear. The mass optimized preferred variation is shown in Fig. 12 (right side).

Fig. 12: Variation study design space: Initial state (left) – space optimized gear (right)
The mass savings amounts in this example to about 25% in respect to the original design. At the same time the optimization of the construction space amounts to 15%, see Fig. 13.

In a second step the consideration of CAD geometry data of housings will be possible. Therefore the software import a standard geometry format, generate Finite Element models, calculate stiffness matrices for the housing and deliver this information to the design process of MDESIGN® gearbox, Fig. 14.

Optimization of micro geometry

The calculation of load distribution in a planetary gear system essentially depends on the helix angel deviation between the contact flanks of the gear pairs. It can be understood as
the sum of different influences. It is assumed that the effects are overlying independently, 
the sum of contact line deviation can be calculated with the single deviations, /11/.

The calculation of single displacements and deformations of all gear box bodies – especially 
the planet carrier, the coupling of ring gear and gear wheel bodies and the deformation of 
teeth – is in planetary gearboxes more complex than in spur gearboxes. To determine the 
load distribution the flank deviation for the tooth contact sun/planet and the tooth contact 
planet/ring gear is calculated by the new software MDESIGN® LVR\textsuperscript{planet}, /8,9/.

The result of the calculation is the excessive of the line load, which is expressed by the factor 
$K_{Hß}$. In general the excessive of the line load is on the flank side opposite to the deviated 
flank side.

Next to the calculation of the ratio of maximum and middle line load the software gives 
detailed information about tooth flank pressure and tooth root stress distribution, Fig. 15.

![Verification of planetary gear stages](image)

Fig. 15: Verification of planetary gear stages

The flank deviation (FLKM) consists of following parts:
- elastic deformation of gear body ($v_{e_RK}$)
- elastic tilting difference of roller bearings /17/ ($v_{e_{WL}}$)
- torsion deformation of planet carrier ($v_{e_{PT}}$)
- tilting of planet because of sliding bearing ($v_{kipp_{PL}}$)
- effective helix angle modification ($f_{Hß_{eff}}$)
- elastic deformation of tooth flank
- elastic deformation difference of planet carrier bearing
- deformation of housing

The helix angle deviation for tooth contact sun/planet is calculated by the following 
equation.

$$FLKM_{1/2} = v_{e_1} + v_{e_{1/2}} + v_{e_{WL1/2}} + v_{e_{PT1/2}} + v_{kipp_{PL1/2}} + f_{Hß_{eff1/2}}$$

The helix angle deviation for tooth contact planet/ring gear is calculated by the following 
equation.

$$FLKM_{2/3} = v_{e_2} + v_{e_{2/3}} + v_{e_{WL2/3}} + v_{e_{PT2/3}} + v_{kipp_{PL2/3}} + f_{Hß_{eff2/3}}$$
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- \( v_e_1 \): deformation difference of sun
- \( v_e_2 \): deformation difference of planet
- \( v_e_3 \): deformation difference of ring gear

The deformation is calculated by the FE-method and afterwards it is added to the flank deviation. All parts of the helix angle deviation have to be added as values normal to the flank. The database of the calculation is saved in XML-Format. With this a structured depositing of design-, modifications-, deviation-, load- and control data is possible. Furthermore the program has a project management which is based on a database to save projects, for standard examples and more calculation guidelines, /14/.

After input of all necessary parameters all data are checked, the design models are generated and the FE-models for the gears with coupling design and the planet carrier are created. For an efficient calculation it is necessary and reasonable to use software. DriveConcepts GmbH develops software solutions for drive technology, which is characterized by clear and intuitive handling of all data. In the background academic established calculation kernels and consistent structured interfaces help to solve the actual task efficiently.

Case study

The example of a wind turbine with 2000 kW output power should show the consequences of different flank modifications with constant load, /12/.

| module | \( m \) | 16 mm | face width | \( b_{1|2|3} \) | 310 mm |
|---------|--------|--------|------------|------------|--------|
| number of teeth \( z_{1|2|3} \) | 20 | 36 | -91 | add. modification sun | \( x_1 \) | 0.4 |
| center of distance \( a \) | 463 mm | add. modification planet | \( x_2 \) | 0.3156 |
| pressure angle \( \alpha \) | 20° | add. modification ring gear | \( x_3 \) | -1,6429 |
| helix angle \( \beta \) | 8° |

Fig. 16: Application case
The main gearbox consists of one planetary gear stage and two spur gear stages (helical gearing). The detailed parameters of the 1st planetary gear stage are listed in Fig. 16. The initial state of unmodified gearing under nominal load is shown in Fig. 17 at left side. In this case the ratio of maximum and mean value of line load is 1.67.

In the first step of optimization with a helix flank modification the factor can be reduced to $K_{H1}=1.23$, see Fig. 17 at right side.

The rest of unbalanced distribution along the face width, which comes from planet carrier torsion deformation, can be compensated with an optimal lead crowning. The ratio of maximum and middle line load can be reduced to $K_{HII}=1.16$, Fig. 18 at left side.

At right side of Fig. 18 is shown that an oversized lead crowning can also lead to poor conditions. In this case the lead load distribution changes to $K_{HII}=1.98$.

The example shows the must of the right dimension of macro geometry and also of used modifications. If these are right the lead load distribution $K_{HII}$ can be reduced from 1.67 to 1.16. But with unfavorable modifications the opposite will be achieved.

This case study shows many advantages of the software MDESIGN® 2010 with the libraries LVR, LVR$^\text{planet}$ and gearbox to develop gearboxes in a very efficiency way.
Reference list

[16] Schulze, Tobias: Calculation of load distribution in planetary gears for an effective gear design process. AGMA Fall Technical Meeting 2010, October 17-19, 2010, Milwaukee Wis, USA